Spatial Variability of the Hurst Exponent for the Daily Scale Rainfall Series in the State of Zacatecas, Mexico

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ABSTRACT

The structural pattern of rainfall data exhibits random fluctuations over time and space. Utilizing concepts of fractal theory, it has been possible to identify characteristics of rainfall data beyond simple statistical indicators of their randomness. The objective of this research was to identify the spatial variation of the Hurst exponent, extracted through standard wavelet techniques from time series of daily rainfall data in the state of Zacatecas, Mexico. The Hurst exponent was extracted for 26 locations using the reference techniques for auto-affine traces—in particular, the wavelets method. Results have shown that the Hurst exponents of rainfall time series are negatively influenced by altitude; thus, stations located at higher altitudes were characterized by Hurst exponents indicating more nonpersistent behavior. The trends among geographical variables (west longitude and latitude) and climatic parameters (annual rainfall and number of rainy days) and their relationship with the Hurst exponent were also analyzed.

1. Introduction

Variations in dynamic and thermodynamic processes cause in the atmosphere nonlinear responses. In dynamic systems, this kind of response generates irregularity, which may show a random pattern of a certain type. To understand the system's irregular pattern for prediction purposes, it is necessary to decide if its dynamic follows a chaotic, random, or deterministic structural pattern (Silva et al. 2006).

Understanding the random patterns of climate is a key aspect in programming preventive activities to better meet the possible occurrences of adverse events that threaten human population or agricultural production systems, such as droughts or floods (Men et al. 2004), as well as to improve physical and biotic resources management (Bullock 2003).

Climate is a dynamic system, and its irregularities or variations are subjected to the influence of stochastic and cyclic factors. Fractal theory is an efficient tool to describe the irregular and complex behavior of dynamic systems (Men et al. 2004).

a. Fractal characterization of rainfall time series

The structural pattern of rainfall data exhibits fluctuations in time and space. Fractal theory has helped to understand the statistical trends of rainfall time series in terms of categorizing their persistence, antipersistence, or chaotic behavior. The established scale invariance of rainfall time series has helped to obtain information even in ungauged areas and to generate databases for forecasting purposes (Olsson et al. 1992; Olsson and Niemczynowicz 1996; Radziejewski and Kundzewicz 1997; Miranda et al. 2004).

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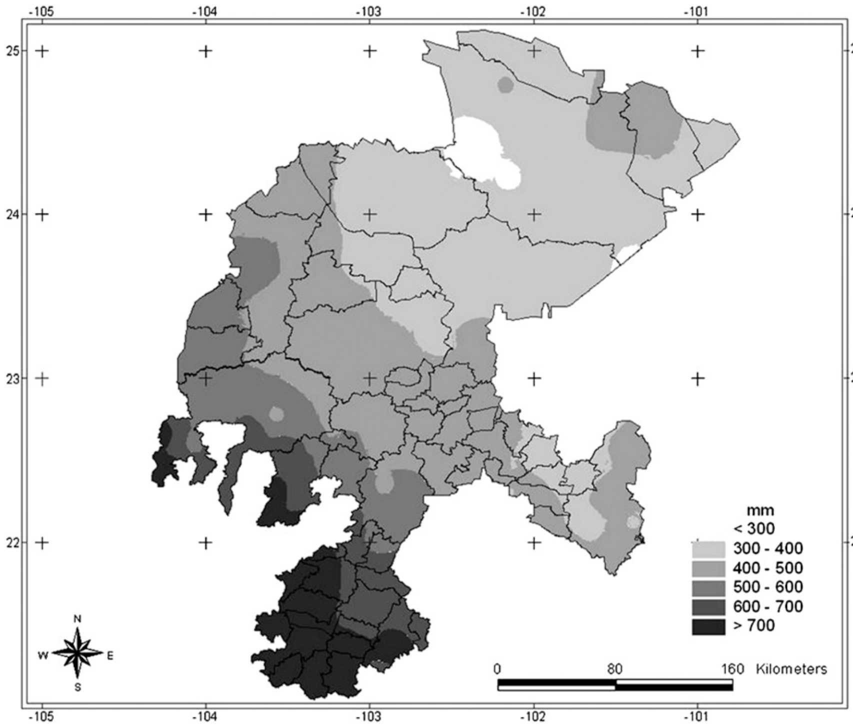

FIG. 1. Spatial variability of the mean annual rainfall in the state of Zacatecas, Mexico (Medina García et al. 2003).

Burgos and Pérez Valdés (1999) used fractal indexes to compare the randomness from several rainfall time series in a decade scale and to obtain an estimation of their normality. In a similar way, Rangarajan and Sant (2004) analyzed Indian climatic dynamics using fractal dimension theory and found that the precipitation during the southwest monsoon is affected by the temperature and pressure variability in the previous winter. At a spatial scale, the fractal patterns of rainfall series are influenced by geographic aspects, whereas their temporal tendencies depend on the local climatic conditions (Kyriakidis et al. 2004).

b. *Environmental variability in the state of Zacatecas*

The state of Zacatecas is located in the north-central region of Mexico (21°09′–25°09′N, 100°47′–104°10′W) and has extended flat, desert lands with valleys inserted among high hilly lands in the northern part, while mountains and steep-slope lands prevail in the southern part, named Los Cañones. The state of Zacatecas has an area of 74 708 km^2 (INEGI 2006) that represents 3.7% of the Mexican territory. Annual rainfall range is from less than 300 to more than 700 mm (Fig. 1). This rainfall gradient results in contrasting vegetation types and high possibilities of natural resources utilization (Medina García and Ruiz Corral 2004).

This extensive spatial variation of the average annual rainfall in Zacatecas is the result of geographic and topographic variables (latitude, longitude, and altitude), producing the driest region in the north and the wettest in the southwest part of the state of Zacatecas (Medina García and Ruiz Corral 2004). Orographically, it is formed by the confluence of four physiographic regions: Sierra Madre Occidental from the western part, Sierra Madre Oriental from the northern and central parts, Meseta Central from the central part, and the trans-Mexican Volcanic Belt from the southeast. The altitudinal range is 2127 m, with the highest site (3200 m) located at the Sierra el Astillero, in the county of Mazapil, and the lowest one (1073 m) at San Agustin, in the municipality of Juchipila. The mean altitude above sea level is 2230 m.

Most of Zacatecas territory has a subtropical arid temperate climate (Medina García et al. 1998); however, some climate variants are subtropical arid semihot (northern regions) and subtropical semiarid temperate (southern and southwestern regions). Mean annual temperature is between 16° and 18°C, and mean annual precipitation is 490 mm, occurring mainly in summer. In some regions of Zacatecas, the rainfall is not enough to satisfy even 50% of the crop's evapotranspiration needs (Mojarro 2004).

TABLE 1. Location, altitude, and period of rainfall time series for weather stations in the state of Zacatecas (from Medina García and Ruiz Corral 2004).

Station	County	Lon (W)	Lat (N)	Alt (m MSL)	Period
Agua Nueva	Villa de Cos	102°09′	23°46′	1932	1963–2003
Camacho	Mazapil	102°22′	24°26′	1658	1961–2003
Cañitas	Cañitas de Felipe Pescador	102°43′	23°46′	1932	1961–2000
Cedros	Mazapil	101°46′	24°20′	1763	1971–2003
Coapas	Mazapil	102°10′	23°50′	2000	1971–2003
Concepción del Oro	Concepción del Oro	102°43′	23°41′	2025	1961–2003
Chalchihuites	Chalchihuites	103°53′	22°38′	2260	1963–2003
El Platanito	Valparaíso	104°03′	23°57′	990	1963–2003
El Sauz	Fresnillo	103°12′	21°46′	2090	1963–2003
La Florida	Valparaíso	103°36′	23°34′	1870	1963–2003
La Villita	Tepechitlán	103°19′	22°56′	1790	1963–2003
Jiménez del Teúl	Jiménez del Teúl	103°47′	22°20′	1900	1963–2003
Juan Aldama	Juan Aldama	103°23′	22°17′	1995	1963–2003
Juchipila	Juchipila	103°06′	23°49′	1270	1963–2003
Monte Escobedo	Monte Escobedo	103°33′	21°46′	2190	1963–2003
Nieves	General Francisco R. Murguía	103°00′	23°17′	1900	1963–2003
Ojocaliente	Ojocaliente	102°16′	21°27′	2050	1963–2003
Pinos	Pinos	101°34′	22°28′	2408	1963–2003
Sain Alto	Sain Alto	103°14′	22°55′	2030	1963–2003
Tecomate	Jalpa	103°02′	21°15′	1375	1963–2003
Trancoso	Guadalupe	102°21′	22°38′	2190	1963–2003
Tlaltenango	Tlaltenango de Sánchez Román	103°17′	23°33′	1700	1963–2003
Villa de Cos	Villa de Cos	102°20′	21°13′	2050	1963–2003
Villa García	Villa García	101°57′	21°12′	2102	1963–2003
Villanueva	Villanueva	102°53′	22°09′	1920	1963–2003
Zacatecas	Zacatecas	102°34′	22°36′	2485	1963–2003

In Mexico, several measures of climate characterizations have been used, taking as the main parameter the coefficient of variation of the rainfall series $CV = \delta/\mu$, where δ is the standard deviation and μ is the mean of the rainfall record. Results have shown a negative correlation between rainfall magnitude and this coefficient, with the highest values observed in the arid portion of the country (Wallén 1955; Mosiño and García 1966; Granados Ramírez et al. 2004; Peralta-Hernández et al. 2008). The objective of this research was to quantify the spatial variation of the randomness of daily rainfall time series in the state of Zacatecas, Mexico, using a fractal approach.

2. Methods

a. Weather stations

The state of Zacatecas has 98 weather stations; out of them, 26 were selected for this research according to the criteria of sufficient data period length (with a minimum of 20 yr of valid information between 1961 and 2003) and representativeness of all parts of Zacatecas (Table 1; Fig. 2). From this record, years with four or more months of missing data were eliminated from the analysis, as well as those years with two or more months with missing data within the rainy season. All selected stations met these requirements except the Cedros and Coapas stations, whose records only started in 1971 and were only included in order to meet the criterion of regional representativeness. For time series analysis, daily records of rainfall were considered in each station, that is, those days without record were included as well. Missing data were substituted with computed ones, utilizing the climate generator ClimGen (Stöckle and Nelson 2003).

b. Fractal analysis

For obtaining the Hurst exponents, each rainfall file was transformed into a time series file (.ts file-name suffix) to be used in the Benoit program [reviewed in Seffens (1999)].[1] With this software, the Hurst exponent can be extracted from the rainfall time series using several reference techniques for self-affine traces; in this research, we only used the wavelets method.

1) THE HURST EXPONENT

The Hurst exponent measures the long-term memory spread of a dataset. According to its value, a time series is classified as persistent ($0.5 < H \leq 1$) or nonpersistent

[1] Benoit is a registered trademark of TruSoft Int'l, Inc.

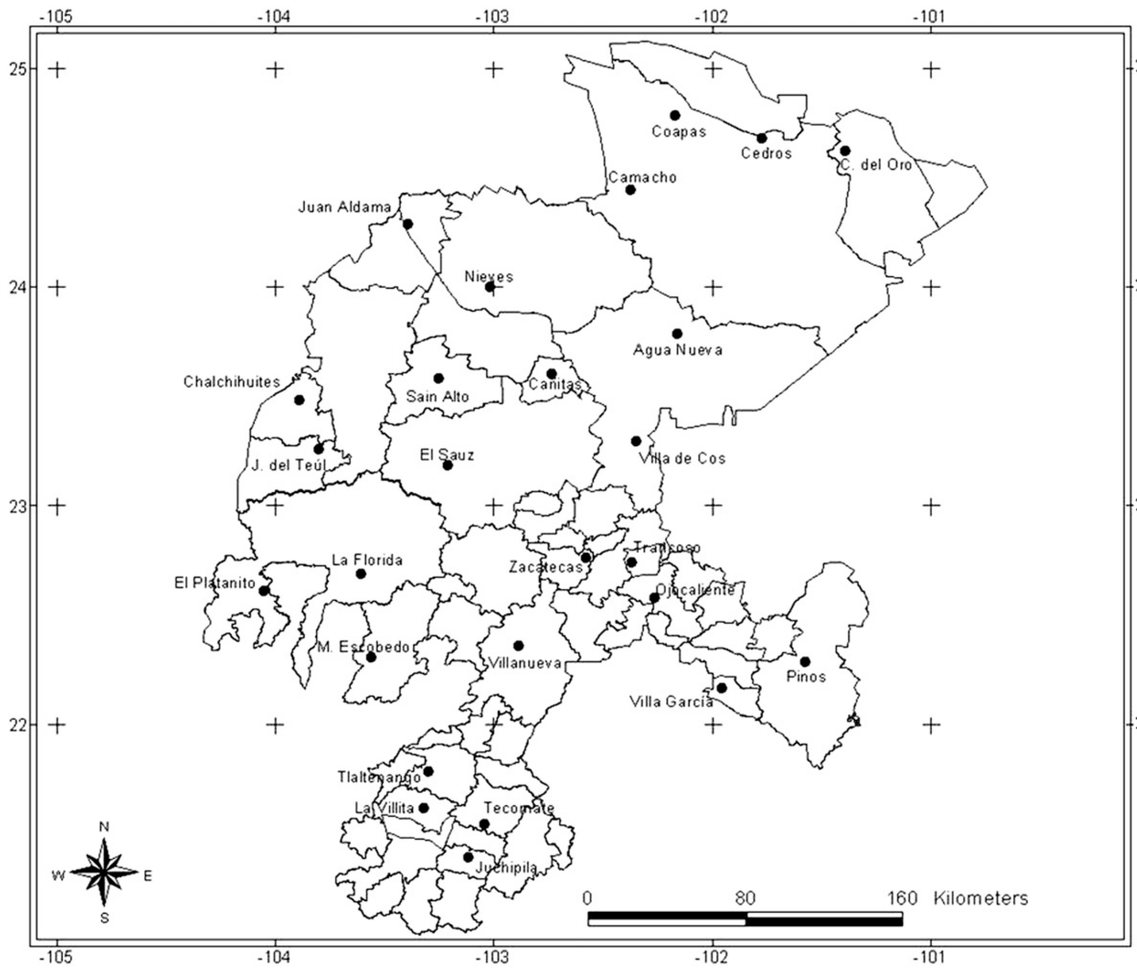

FIG. 2. Spatial distribution of weather stations in the Zacatecas state.

($0 \leq H < 0.5$). If $H = 0.5$, then the subsequent data are not intercorrelated, meaning that the future values of the time series are not influenced by the present or past values (Palomas 2002; Sakalauskienė 2003); those series are classified as unpredictable. This last case corresponds to the white noise or the classic Brownian movement. The two former cases describe fractional Brownian movements.

The Hurst exponent H of a real-valued time series $\{Z_1, Z_2, \ldots, Z_n, \ldots\}$ is defined as the exponent in the asymptotic scaling relation

$$\left\langle \frac{R(n)}{S(n)} \right\rangle = Cn^H \quad \text{as} \quad n \to \infty, \tag{1}$$

where C is a constant, angular brackets denote expected value, $S(n)$ is the standard deviation of the first n data of the series $\{Z_1, Z_2, \ldots, Z_n\}$, and $R(n)$ is their range:

$$R(n) = \max\{Z_1, Z_2, \ldots, Z_n\} - \min\{Z_1, Z_2, \ldots, Z_n\}.$$

In the Benoit software, the Hurst exponent H can be directly found using the definition in Eq. (1) (this is the rescaled range technique and provides the estimate $H_{R\text{or}S}$); H can also be computed from the power spectrum of the time series $\{Z_1, Z_2, \ldots, Z_n, \ldots\}$ (the spectral method gives an estimate H_S) or using wavelet techniques (yielding an estimate H_w). The last technique is the most popular in fractal rainfall studies (Rehman and Siddiqi 2009), and we have also selected it for our research.

2) THE WAVELETS METHOD ESTIMATE OF THE HURST EXPONENT (H_w)

The wavelets method is valid for self-affine traces, where the variance is not constant as the window size increases. If $f(t)$ is a self-affine random process, t is a position parameter (time or distance), $a > 0$ is a scale (dilation) parameter, $w(t)$ is a mother wavelet, and $w_{t,a}(t') = (1/\sqrt{a})w(t' - t)/a$ is its shifted, dilated, and scaled version, then the continuous wavelet transform of $f(t)$ is defined as

TABLE 2. Basic statistics of rainfall time series for each weather station in the state of Zacatecas. Range is maximum minus minimum daily value. Abbreviations: Y, years of information; M, average; SD, standard deviation; VC, variation coefficient; S, skewness; and K, kurtosis.

Station	County	Y	Range	M	SD	VC	S	K
Agua Nueva	Villa de Cos	34	52	0.96	3.87	4.03	6.30	49.74
Camacho	Mazapil	31	62	0.74	3.28	4.41	6.99	65.55
Cañitas	Cañitas de Felipe Pescador	27	68	0.97	4.04	4.17	6.75	59.57
Cedros	Mazapil	30	90	0.92	3.71	4.03	7.09	80.65
Coapas	Mazapil	39	92	1.40	4.72	3.38	5.53	44.67
Concepción del Oro	Concepción del Oro	31	67	1.06	4.12	3.88	6.39	54.58
Chalchihuites	Chalchihuites	29	83.1	1.23	4.39	3.57	6.41	58.70
El Platanito	Valparaíso	40	108.5	1.61	5.63	3.50	5.81	47.13
El Sauz	Fresnillo	40	67.2	1.15	4.23	3.68	5.94	46.57
La Florida	Valparaíso	35	77	1.35	4.48	3.32	5.53	44.72
La Villita	Tepechitlán	36	80	1.15	4.83	4.20	6.70	60.44
Jiménez del Teúl	Jiménez del Teúl	35	98	1.99	6.29	3.15	4.98	35.11
Juan Aldama	Juan Aldama	41	85	1.60	5.26	3.29	5.19	36.26
Juchipila	Juchipila	40	82.1	2.17	6.58	3.04	4.61	26.78
Monte Escobedo	Monte Escobedo	38	70	1.97	5.86	2.98	4.56	26.56
Nieves	General Francisco R. Murguía	39	83	1.02	4.32	4.25	7.28	74.42
Ojocaliente	Ojocaliente	39	87	1.09	4.45	4.10	6.90	66.37
Pinos	Pinos	31	80.7	1.14	4.87	4.27	6.85	61.38
Sain Alto	Sain Alto	35	91	1.27	4.60	3.61	6.14	53.29
Tecomate	Jalpa	32	76.5	1.82	5.70	3.14	4.76	29.13
Trancoso	Guadalupe	38	69.5	1.22	4.73	3.89	6.24	50.12
Tlaltenango	Tlaltenango de Sánchez Román	37	77.5	1.91	5.92	3.10	4.71	28.78
Villa de Cos	Villa de Cos	36	86	1.10	4.72	4.30	6.78	59.73
Villa García	Villa García	39	82.1	1.21	4.97	4.13	6.86	63.40
Villanueva	Villanueva	32	53.7	1.30	4.38	3.35	4.91	30.34
Zacatecas	Zacatecas	39	75	1.34	5.00	3.72	5.83	43.95

$$W(t,a) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} w_{t,a}(t') f(t')\, dt'. \quad (2)$$

If the time series $f(t)$ is self-affine, the variance of $W(t,a)$ will scale with the dilation parameter asymptotically as

$$V(a) = \langle W^2 \rangle - \langle W \rangle^2 \propto a^{\delta}. \quad (3)$$

The exponent δ is between -1 and $+3$, $-1 \le \delta \le 3$. The Hurst exponent is defined as

$$H_w = \begin{cases} \dfrac{\delta+1}{2} & \text{if} \quad -1 \le \delta < 1 \quad \text{(FGN)} \\ \dfrac{\delta-1}{2} & \text{if} \quad 1 \le \delta \le 3 \quad \text{(FBM)} \end{cases}, \quad (4)$$

where FGN is fractal Gaussian noise and FBM is fractional Brownian motion.

The numerical algorithm in Benoit, based on the theoretical scaling law [Eq. (3)], considers n wavelet transforms, each with its own different scaling coefficient a_i. Let $S_1, S_2, \ldots, S_n$ be, in turn, the standard deviations from zero for the wavelet transforms with the respective scaling coefficients (a_i, $i = 1, 2, \ldots, n$).

Define the ratios of standard deviations $G_1, G_2, \ldots, G_{n-1}$ as

$$G_1 = \frac{S_1}{S_2}, \quad G_2 = \frac{S_2}{S_3}, \ldots, G_{n-1} = \frac{S_{n-1}}{S_n} \quad (5)$$

and take the average value of G_i as

$$G_{\text{avg}} = \frac{1}{n-1} \sum_{i=1}^{n-1} G_i. \quad (6)$$

In the Benoit software, the Hurst exponent is calculated as

$$H = f(G_{\text{avg}}), \quad (7)$$

where f is a heuristic function that has been found to approximate well the Hurst exponent in the form $H_w = f(G_{\text{avg}})$ for self-affine stochastic traces. The Benoit package sets $n = 4$ and $a_i = 2^i$ for $i = 0, 1, 2, 3$. The mother wavelet used is a step function.

As is well known (Carbone et al. 2004), the Hurst exponent is linked with fractal dimension D as

$$H = 2 - D. \tag{8}$$

Rangarajan and Sant (2004) and Rehman and Siddiqi (2009), who used this wavelet-based method to find $H(=H_w)$ for Indian and Saudi Arabian rainfall data, respectively, defined the climate predictability index as

$$\mathrm{PI} = 2|D - 1.5| = 2|0.5 - H|. \tag{9}$$

If PI is close to 0, they claim that the climate is unpredictable; the closer it is to 1, the more predictable it is.

c. *Descriptive statistics*

From the rainfall data, we also computed basic statistics (standard deviation, variance, and coefficient of variation), the coefficient of asymmetry, and kurtosis as complementary results in order to characterize rainfall time series by means of descriptive statistics and to relate these statistics to fractal parameters.

3. Results and discussion

a. *Basic statistics*

Statistics for rainfall time series for all weather stations are shown in Table 2. Large differences in minimum and maximum values were detected among the sites, the extreme case being the station El Platanito, where a precipitation event of 108.5 mm was registered; however, in the other weather stations, the range between minimum and maximum values was acceptable. Standard deviation ranged between 3.28 and 6.58 mm; however, since average daily precipitation is too low (between 0.74 and 2.17 mm day^{-1}), the coefficient of variation homogenizes the variation among stations. The values of asymmetry indicated that most of the rainfall data have positively skewed distribution. The degree of peakedness (kurtosis) of rainfall distribution was greater than three in all stations, which indicates that all distributions are leptokurtic (Haan 1979).

b. *Fractal analysis*

Table 3 shows the value of the Hurst exponents [and the predictability index; see Eq. (9)] for all stations. Hurst exponents less than 0.5 were estimated for all stations, with an average value of 0.10 and values ranging from 0.02 to 0.30 with a high coefficient of variation (72.6%). In general, the temporal pattern of the rainfall series in Zacatecas shows antipersistent behavior, that is, a negative dependence between long-separated events. As reported from Venezuela (Amaro et al. 2004) and, very recently, from central Mexico (Valdez-Cepeda et al. 2012), the degree of antipersistence in some cases depends on the time scale: if the Hurst analysis were done for monthly or annual time series, the Hurst exponent would have possibly exceeded 0.5, indicating a tendency for persistence. If H is scale dependent, this implies multifractal behavior and would necessitate multifractal wavelet analysis [in the spirit of Davis and Wiscombe (1994)]. In an influential paper, Kalisky et al. (2005) [see also Kalisky et al. (2007)] recommended a so-called volatility test for a ‘‘quick look’’ decision on whether a time series is fractal or multifractal. A careful analysis of these papers [see, e.g., Eq. (13) in Kalisky et al. (2007)] shows that the basic tenet behind the proposed volatility analysis is the validity of the following assumption for four identically distributed random variables [the so-called four-point theorem of nonlinear signal processing (Bendat 1981) or Wick's (1950) theorem from quantum field theory]:

$$\langle ABCD\rangle = \langle AB\rangle\langle CD\rangle + \langle AC\rangle\langle BD\rangle + \langle AD\rangle\langle BC\rangle, \tag{10}$$

TABLE 3. Hurst exponent variability for the precipitation time series in the state of Zacatecas.

Station	County	H_w	$\mathrm{PI} = \lvert 0.5 - H_w\rvert$
Agua Nueva	Villa de Cos	0.04	0.92
Camacho	Mazapil	0.02	0.96
Cañitas	Cañitas de Felipe Pescador	0.02	0.97
Cedros	Mazapil	0.03	0.94
Coapas	Mazapil	0.05	0.90
Concepción del Oro	Concepción del Oro	0.10	0.80
Chalchihuites	Chalchihuites	0.09	0.82
El Platanito	Valparaíso	0.15	0.70
El Sauz	Fresnillo	0.12	0.76
La Florida	Valparaíso	0.21	0.58
La Villita	Tepechitlán	0.30	0.40
Jiménez del Teúl	Jiménez del Teúl	0.11	0.78
Juan Aldama	Juan Aldama	0.10	0.80
Juchipila	Juchipila	0.22	0.56
Monte Escobedo	Monte Escobedo	0.19	0.62
Nieves	General Francisco R. Murguía	0.08	0.84
Ojocaliente	Ojocaliente	0.07	0.86
Pinos	Pinos	0.05	0.90
Sain Alto	Sain Alto	0.04	0.92
Tecomate	Jalpa	0.25	0.50
Trancoso	Guadalupe	0.12	0.76
Tlaltenango	Tlaltenango de Sánchez Román	0.21	0.58
Villa de Cos	Villa de Cos	0.02	0.96
Villa García	Villa García	0.05	0.90
Villanueva	Villanueva	0.10	0.80
Zacatecas	Zacatecas	0.06	0.88

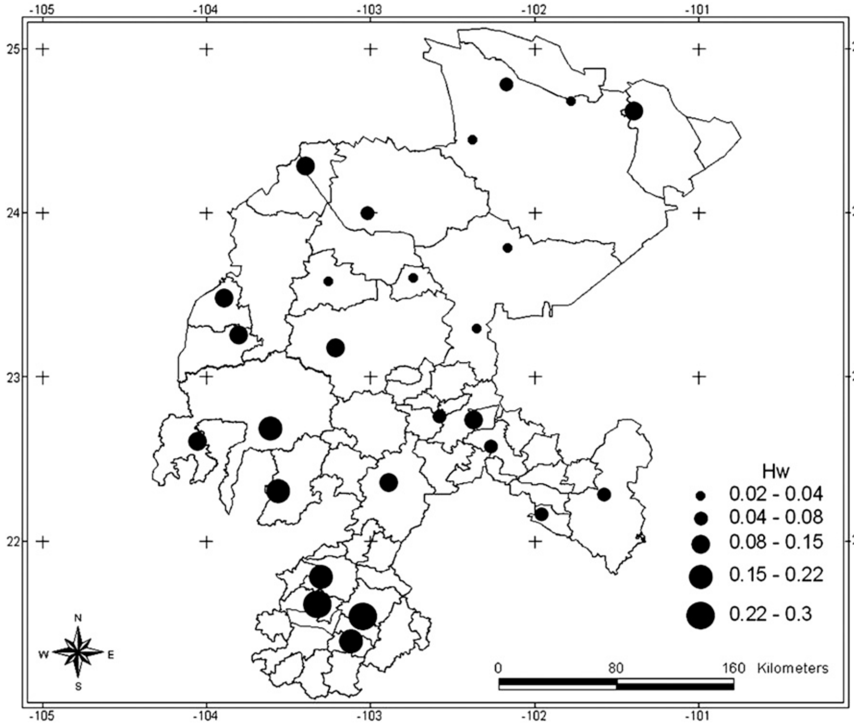

FIG. 3. Spatial variability of the wavelet Hurst exponent H_w for the rainfall time series in the state of Zacatecas.

where angular brackets are expectancies. Applying Eq. (10) to the case $A = B = C = D = X - \langle X \rangle$, where X is a random variable, we find that the kurtosis of X must be 3, which is certainly not true for our Mexican rainfall data, which are strongly leptokurtic! Even if the quick-look volatility analysis is not necessarily applicable to our leptokurtic data, the results of Amaro et al. (2004) and Valdez-Cepeda et al. (2012) warn us of a real possibility of multifractality, that is, of a possible time and duration dependence of H_w. To check this, in future research the rainfall data should be subjected to multifractal analysis. As pointed out by a reviewer of an earlier version of the paper, even in the case of shorter daily or weekly data ranges, a careful comparison of the wavelet transforms with increasing dilatation constants a_i might give indication of the fine details of rainfall dynamics.

Within each rainfall time series, days without rain may be influencing the value of the Hurst. Nevertheless, within the rainy season in each station the midsummer drought may be responsible for the antipersistent behavior of the series. This may be the case of the stations north of the state, where the rainfall pattern shows unimodal behavior, which is typical of Mexican monsoon. The midsummer drought may cause antipersistent behavior of rainfall mainly in those years where this phenomenon is more evident. This applies to the weather stations northward of Zacatecas, where, at annual scale, a unimodal-type rainfall distribution is observed, which is typical of the Mexican monsoon (Magaña et al. 1999, 2003; Peralta-Hernández et al. 2008).

This antipersistent behavior within time series should be incorporated in modeling processes in Zacatecas when a climate generator will be used, given that at daily scales, the structural patterns of rainfall time series are different between northern and southern regions in the state. Spatial variability of persistence measured by the Hurst exponent and extracted from each rainfall series is shown in Fig. 3.

As a validation of these results, two different weather stations were considered: Parras in the state of Coahuila in the northern part of Mexico and El Tule, Arandas, in the state of Jalisco in the central part of Mexico. The temporal variations of the time series (including 365 days) of daily precipitation for both weather stations are presented in Fig. 4. The Hurst exponents for the two stations were 0.079 and 0.262, respectively. For arid lands, the main characteristic is the occurrence of individual storms when the time series shows high variability ($H_w = 0.079$); this type of behavior in time series is caused by the differences in daily precipitation among events. These results are similar to those of Wallén (1955) and Mosiño and García (1966),

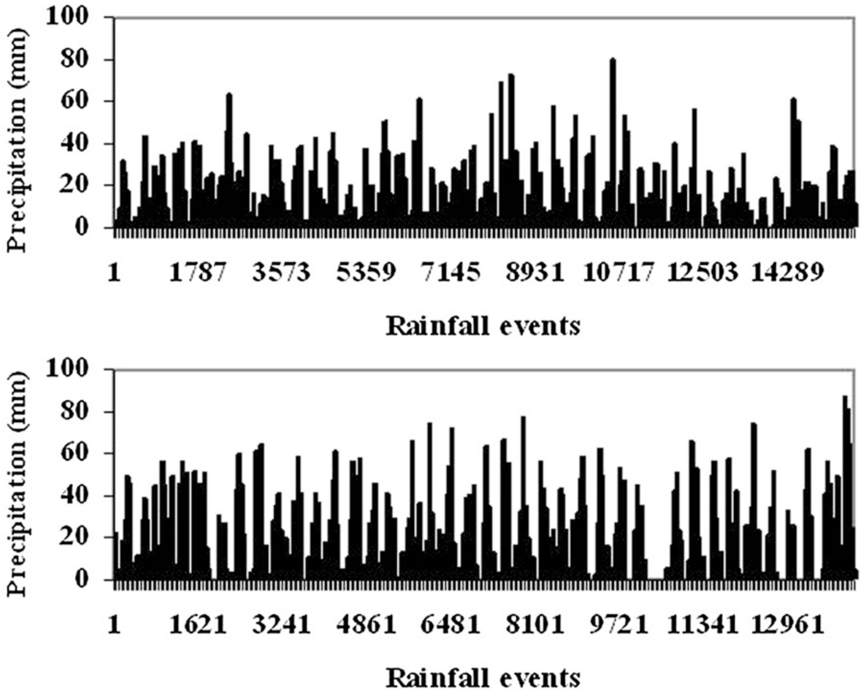

FIG. 4. Daily precipitation time series for (top) Parras and (bottom) El Tule stations.

who also obtained high coefficients of variation for arid regions.

The main advantage of using Hurst exponents rather than the coefficient of variation is that the former is a numerical representation of the randomness through the history of a process that is a dynamical characteristic, while the latter is a statistic that is independent of the temporal evolution.

Time series with more rainfall events and more similitude among them presented a higher Hurst exponent ($H_w = 0.262$). This means that time series like at El Tule site have a higher potential to predict a pattern or behavior of daily rainfall than at the Parras site.

c. *Relationship between the Hurst exponent and geographical parameters*

Temporal behavior of rainfall time series may be regionalized in space in order to study its relation with geographical and physiographical variables (Kyriakidis et al. 2004; Miranda et al. 2004). It might be expected that the effect of altitude on the dynamics of rainfall series is especially strong. Actually, the results showed that there is no relationship between annual rainfall average and altitude ($r = 0.12$), and the Hurst exponent of the time series is also weakly influenced by altitude ($r = 0.03$).

The spatial variability of the Hurst exponent of rainfall time series has been documented in this research mostly based on western longitude coordinates of the weather stations (Fig. 5). An inverse correlation between persistence (in space and time) and latitude has been reported for rainfall (Miranda et al. 2004); however, this has been documented for tropical conditions only, which are very different from the environments studied in this research. It should be noted that for semiarid to arid conditions that prevail in the state of Zacatecas, the "statistical roughness" of rainfall series (as measured by the Hurst exponent) is higher in northern areas as long as the western longitude degrees decrease ($r = 0.57$), as shown Fig. 5. In this case, the antipersistence

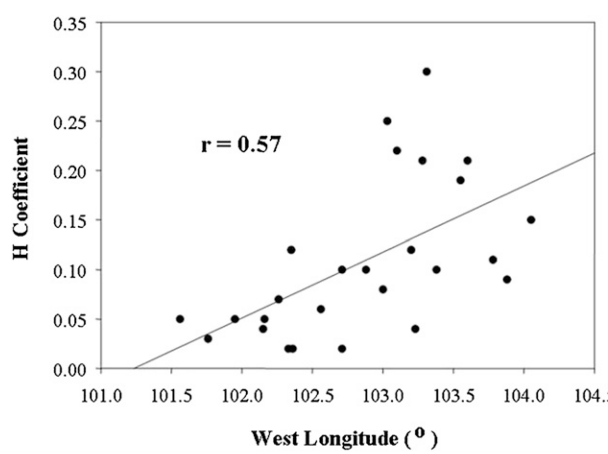

FIG. 5. Effect of geographical (longitude) location on rainfall time series Hurst exponents in the state of Zacatecas.

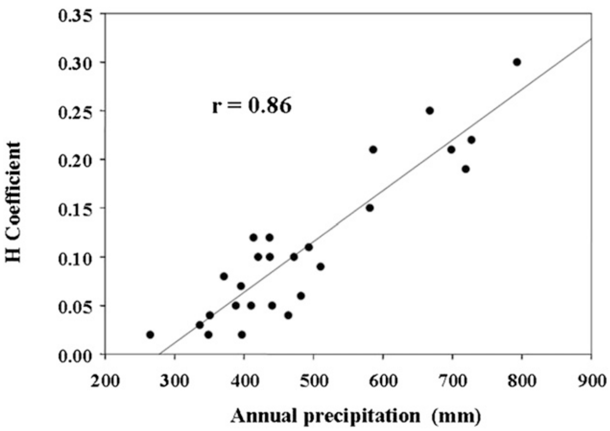

FIG. 6. Annual rainfall and Hurst exponent relationship for the weather stations in the state of Zacatecas.

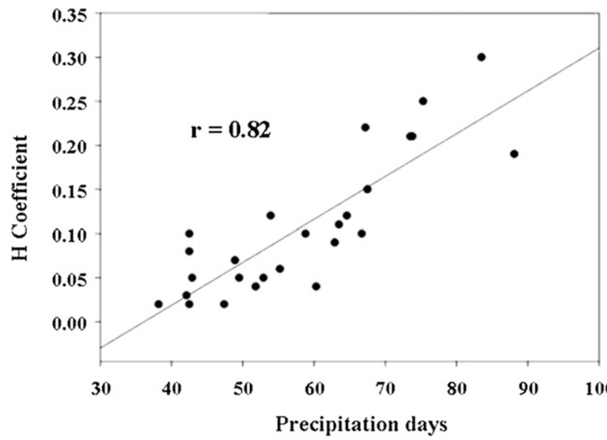

FIG. 7. Relation between number of precipitation days and the Hurst exponent in the state of Zacatecas.

that is present in rainfall series seems to be associated with the magnitude of rainfall events and their temporal distribution and not with the frequency of the rainfall.

Less roughness of rainfall data is observed in the southwestern region. This behavior is typical in sites with larger yearly rainfall volume and is influenced at a mega scale by topography. This is shown in Fig. 6, where the relationship between annual rainfall and Hurst exponent is evident ($r = 0.86$).

Another important parameter that determines the structural pattern in rainfall time series is the number of rainy days. In this research we found a strong correlation ($r = 0.82$) between the number of rainy days and the Hurst exponent (Fig. 7). Both relationships (in Figs. 6, 7) have positive slope, indicating higher Hurst exponents at sites with higher average precipitation values.

We find Figs. 6 and 7 very difficult to explain, as the Hurst exponents H_w had been determined from daily time series, which have (apparently) nothing to do with the number of rainy days in a year or total precipitation in a year. Still, the correlations are convincing, and it remains a challenging research task to construct a (multi?) fractal model to satisfactorily explain this empirical finding.

4. Conclusions

Results obtained in this research have shown that all daily rainfall time series studied have antipersistent behavior. The Hurst exponent extracted from rainfall time series is a useful numerical measure that can be used to explain the spatial behavior of the randomness of rainfall in contrasting climatic environments.

In modeling studies of soil erosion processes, it is necessary to consider the gradients of the spatial changes in the Hurst exponent of the daily rainfall time series across the Zacatecas state. Hurst exponents may also be useful for determining the impact of climate change on local precipitation records. Exponents extracted from recent data (e.g., the last 20 yr) may be compared to those from longer periods (e.g., 50 yr or more) to determine the impact of long-range climatic variations.

We assumed that the studied rainfall time series are monofractal, though there are indications (Amaro et al. 2004; Valdez-Cepeda et al. 2012) that this might not be always true. Further research is needed to check the possible multifractality of rainfall data, and, if needed, extract their Hurst exponents with multifractal techniques.

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